Maths for Computing
Tutorial 4

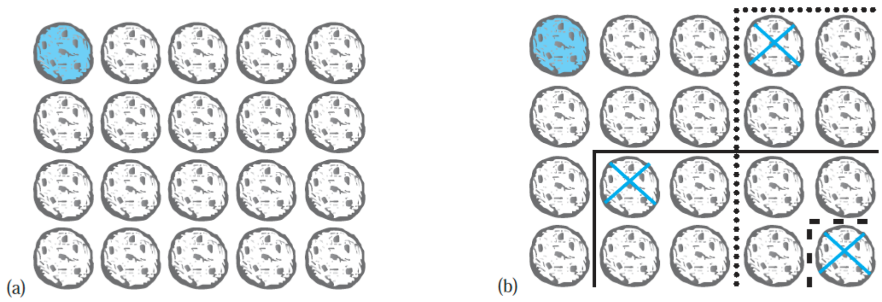
1. Suppose $a, b \in \mathbb{R}$. Prove that if a is rational and ab is irrational, then b is irrational.

2. Prove that $\sqrt{3}$ is irrational.

3. Write the numbers $1,2,3,\ldots,2n$ on a board, where n is an odd integer. Pick any two of the numbers, j and k, write $|j-k|$ on the board and erase j and k. Continue this process until only one integer is written on the board. Prove that this integer must be odd.

4. Prove that there is no positive integer n such that $n^2 + n^3 = 100$.

5. Chomp is a game played by two players. In this game, cookies are laid out on a rectangular grid . The cookie in the top left position is poisoned, as shown in below left figure. The two players take turns making moves; at each move, a player is required to eat a remaining cookie, together with all cookies to the right and/or below it (see below right figure, for example). The loser is the player who has no choice but to eat the poisoned cookie. We ask whether one of the two players has a winning strategy. That is, can one of the players always make moves that are guaranteed to lead to a win?



6. Prove that if the first 10 positive integers are placed around a circle, in any order, there exist three integers in consecutive locations around the circle that have a sum greater than or equal to 17.